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MECHANICS OF STREAMS WITH MOVABLE BEDS OF FINE SAND

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MECHANICS OF STREAMS WITH MOVABLE BEDS OF FINE SAND

Norman H. Brooks,[1] J. M. ASCE

SYNOPSIS

A laboratory study was made of the characteristics of streams flowing over a loose bed of fine sand in order to determine what factors govern the equilibrium rate of transportation of fine sand in suspension. Twenty-two experimental runs were performed in a 40-foot tilting flume for various conditions, with bed sand of two different sizes (0.10 mm and 0.16 mm). Each run represented a uniform open-channel flow in equilibrium with the sand bed.

Because of the extreme variability of channel roughness, the transportation rate could not be expressed as a unique function of the bed shear stress, the channel geometry, and properties of the sand. This is contrary to previous theories for the equilibrium transportation rate of suspended load. At low velocities, the large irregular dunes which formed on the stream bed made the bed friction factor over six times larger than the friction factor for the smooth sand beds obtained at higher flow rates.

By using the mean velocity and the depth (or the water discharge and sediment discharge) as <u>independent</u> variables, and slope as a <u>dependent</u> variable, an orderly qualitative relationship between the pertinent variables was obtained.

INTRODUCTION

The transportation of sediment in suspension in a stream is caused by turbulent diffusion of material upward from the bed. The distribution of the concentration over the depth may be derived by assuming similarity between the suspended sediment transfer and the momentum transfer, and by using von Karman's logarithmic velocity law to obtain the distribution of the diffusion coefficient. The resulting expression, which is called the suspended load equation may be written as follows, according to Ismail:[2]

$$\frac{c}{c_a} = \left(\frac{d-y}{y} \frac{a}{d-a} \right)^z \qquad (1)$$

wherein c is the point concentration of suspended sediment at a distance y above the bed, c_a is the concentration at some reference level y = a, d is the total depth, and z, the exponent, is given by

1. Asst. Prof. of Civ. Eng., California Inst. of Technology, Pasadena, California.
2. "Turbulent Transfer Mechanism and Suspended Sediment in Closed Channels," by H. Ismail, <u>Trans.</u> ASCE, Vol. 117 (1952), pp. 409-446.

$$Z = \frac{w}{\beta k u_*}$$

Here w is the settling velocity of the particles, β is the ratio of the diffusion coefficient for sediment to the kinematic eddy viscosity, k is the von Karman universal constant and u_* is the shear or friction velocity. The derivation of Eq. 1 was first published by Rouse[3] with the simplification $\beta = 1$ and was verified by the experimental work of Vanoni[4].

This suspended load equation is quite satisfactory except near the boundaries where the assumptions under which the equation was derived break down. Indeed, from the equation itself it is easily seen that as $y \rightarrow 0$, $c \rightarrow \infty$, a physically impossible situation. Consequently, it is not a simple matter to supply a boundary condition, and Eq. 1 of necessity contains an unpredictable quantity c_a at an arbitrary reference level $y = a$. Thus Eq. 1 yields only the relative distribution within the stream, whereas the absolute concentrations depend on the diffusion mechanisms right at the sand bed, i.e. the source of the suspended material. To date, little is known about the interactions between a turbulent stream and a movable sand bed. Obviously, if only the relative concentrations are known, it is impossible to find the sediment discharge by integration.

To show how limited the development of suspended sediment transportation theories has been, the problem might be roughly compared to the problem of turbulent flow in pipes. It would be as if only the relative velocity distribution were known for a given pipe size and hydraulic gradient, without any knowledge of how the total discharge is related to the other variables.

The purpose of this paper is to report some experimentally determined relationships between the sediment discharge and the hydraulic characteristics of an open channel flow with a movable bed of fine sand. No attempt is made to derive a general method for obtaining a boundary condition for the suspended load equation, but the shortcomings of two existing theories will be pointed out.

Apparatus and Procedure

Tilting Flume

The experiments were performed in the 40-foot closed-circuit tilting flume shown in Fig. 1. The water was recirculated with its entire sediment load, so that it was not difficult to maintain a stable equilibrium condition for hours at a time. Since the velocity in the return pipe was always high enough to prevent any significant deposition of sand, the layer of sand covering the bottom of the flume contained practically all the sand in the system. The inside surfaces of the flume were painted and were found to be hydrodynamically smooth; no artificial roughness was used.

Because it was found that the temperature had an appreciable effect on the transportation rate, four 1000-watt immersion heaters were installed to regulate the temperature of the water for the second series of runs (Runs 21

3. "Modern Conceptions of the Mechanics of Fluid Turbulence," by H. Rouse, Trans. ASCE, Vol. 102 (1937), p. 534.
4. "Transportation of Suspended Sediment by Water," by V. A. Vanoni, Trans. ASCE, Vol. III (1946) pp. 67-133.

through 30). After Run 21, damping screens were used at the inlet to reduce the large scale turbulence generated in transition section and elbows. The distance required in the flume for establishment of uniform conditions was thereby shortened to generally less than 6 ft. A comparison of Runs 21 and 21a (without and with screens respectively) showed that the final equilibrium established was not affected by the screens.

Procedure for Measurements

The slope of the flume was determined directly from a gage mounted on the truss near the jack used for tilting the flume. A point gage mounted on a carriage was used to measure water surface and bed elevations relative to the flume at a number of stations along the flume. When the water surface was wavy the mean elevation was determined by averaging the point gage readings on adjacent crests and troughs. In order to determine the average elevation of the surface of the sand bed, the bed configuration for the entire flume (starting with Run 12) was leveled in reaches of 2 to 4 ft with a specially built scraper, after stopping the flow of water at the end of each run. The mean depth d was found by averaging the differences between the water surface and bed elevations over the section of the flume in which the flow was in equilibrium. This procedure is illustrated in Fig. 2 for Run 27. For Runs 5, and 8 through 13, the probable error in the measured mean depth is about 0.005 ft, whereas in all the other runs it is of the order of 0.001 ft.

The slope of the energy line relative to the flume was calculated from the bed and water surface measurements, and was added to the slope of the flume to obtain the true energy gradient. The flume was adjusted to keep the relative slope 0.0001 or less. Fig. 2 also shows the calculated energy line for Run 27.

The discharge was determined from the pressure drop across a standard bell-shaped 6 by 4-inch pipe reducer just downstream from the pump (Fig. 1). Since the location was not very desirable, the meter was calibrated with detailed point velocity measurements in the open channel, both with and without sediment, and the venturi coefficient was found to be 0.95. The probable error in the measured discharges are believed to be 1% or less.

The total sediment discharge was measured by sampling the flow in the end tank over the pump with a siphon. The sampler was a vertical copper tube of 0.302-in. inside diameter, with a 180° turn on the bottom end. The entire load is in suspension and well mixed at the sampling position; nevertheless, the sampler tip was moved around gently during sampling to ensure getting representative samples. The head on the siphon was adjusted to make the velocity in the sampler tip equal to the mean velocity of flow. (The fall velocity of the sand was much smaller than the vertical flow velocity in the end tank). In addition, precautions were taken to avoid errors due to sand storage in the sampling tube. For each run a number of one liter samples were taken, usually in groups of three, and the concentration for each sample was determined by filtering the sample, and drying and weighing the residue of sand. For simplicity the concentrations are all reported in grams of dry sand per liter of mixture.

The sediment discharge concentration $\bar{C}$, measured in this way, is thus the total rate of sediment transportation (including bed load) divided by the discharge, and is not necessarily the same as the average concentration of sediment in suspension in the open channel.

For each liter sample withdrawn from the flume, a liter of clear water was added at the same time to keep the depth of flow constant. Since the flow

circuit is closed, the depth in the open channel is governed by the amount of water in the system.

For some of the runs, velocity and concentration profiles were measured in the flume on the centerline. But for many of the runs the bed was covered with large irregular dunes, which moved downstream and deformed continuously, making it practically impossible to measure representative velocity and concentration profiles. Thus good profiles could be obtained only for the runs for which the bed of sand was smooth. The measurements and results are not reported herein owing to lack of space, but are given in detail in the author's thesis.[5]

The laboratory studies also included a study of the mechanism of dune height, wavelength, velocity, propagation, and both still and moving pictures. The findings will be summarized very briefly under "Experimental Results," but a full description including numerous photographs may also be found in the author's thesis.[5]

Sand Characteristics

Fortunately, the uniform sands used by Vanoni[4] and Ismail[2] for suspended load investigations were still available and were used in these experiments, making it unnecessary to prepare new sand. Two different uniform quartz sands were used: Sand No. 1 with mean sedimentation diameter of 0.16 mm for Runs 2 to 13, and Sand No. 2 with sedimentation diameter 0.10 mm for Runs 21 to 30. The sieve analyses of the two sands are shown in Fig. 3 with the geometric mean sieve size (D_g) and geometric standard deviation (σg) indicated for each. At 25°C the mean fall velocities are 0.060 and 0.030 ft/sec respectively. About 145 lb of sand (dry weight) was used in the flume for each of the two series, which was equivalent to a uniform layer slightly over 1/2 in. thick on the bed of the flume.

Establishing Uniform Flow in Equilibrium; Reproducibility

Before making the final measurements for a run it was first necessary to establish uniform flow in equilibrium with its sand bed. With a movable bed this was far from easy because of the changing configurations of the bed (smooth or rippled), and the often-observed tendency for the sand to spread itself unevenly throughout the length of the flume. Equilibrium as used hereafter implies (1) that the mean depth of flow stays constant over a working section of the flume, and (2) that the pattern and distribution of sand on the bed has stopped changing.

Originally, it was believed that the rate of transport and the flow velocity would be uniquely determined by the size of sediment, the size of channel, and the shear stress on the bed, which in turn depends simply on the depth, width, and slope of the stream. Although this belief is commonly held by investigators of sediment transportation phenomena, nevertheless it was found that the relationship is not unique.

This false assumption led to immediate difficulty in programming the runs to be made in the flume. The three main quantities which could be regulated were depth, d, discharge, Q, and slope S. The width is fixed and the sediment

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5. "Laboratory Studies of the Mechanics of Streams Flowing Over a Movable Bed of Fine Sand," thesis by N. H. Brooks, submitted to the California Institute of Technology, Pasadena, Calif., 1954, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

load cannot be controlled directly. At first, an attempt was made to select the depth and slope for each run, and then adjust the discharge to obtain equilibrium; but in the course of the experiments it was found that for some depth-slope combinations, there are two, or maybe even more, different equilibrium discharges and sediment loads. These are possible because of the tremendous variations in the roughness of the channel due to the appearance of dunes of various sizes.

Another source of difficulty with choosing d and S first was the sensitivity of the flow to changes in Q. Small changes in Q, made to adjust to a given slope, often produced significant changes in the other variables, such as sediment discharge and bed configuration. Sometimes it was not even possible to determine immediately whether or not an adjustment or dischage was toward or away from equilibrium. Consequently many hours could be spent trying to obtain an equilibrium by adjusting the discharge to match a given depth-slope combination.

On the other hand, by fixing the rate of flow and the depth, it was possible to make the final adjustment in the slope quite simply. If the first approximation for the slope was in error, the amount of adjustment required could be figured by measuring the slope of the energy line relative to the flume. The adjustment of the slope of the flume would then cause only slight changes in the equilibrium. The slope of the energy line was really already fixed by the flow, so in effect the inclination of the flume itself would simply be adjusted to this slope, with the only net change to the flow being very minor redistribution of depths over the length of the flume.

The time required to reach equilibrium depended on the sediment load and the amount of adjustment of the bed configuration required. With high loads, less than one-half hour was required, but with light loads up to four hours was sometimes needed. The final configuration of the bed was found to be independent of the initial bed configuration, whether flat or in dunes, evenly or unevenly distributed in the flume. The initial condition affected only the amount of time required to reach equilibrium.

The reproducibility of the runs was excellent. This was well demonstrated in Run 28, where two water surface and bed profiles were taken on two different days. Scraping the bed for the first bed elevation profile erased the dunes and ripples, so that the dunes had to be regenerated and equilibrium reestablished before any other measurements, including the second set of profiles, could be made. The two sets of profiles were practically identical, indicating that the equilibrium achieved was stable, well-defined, and reproducible. The calculated mean depths, velocities, and slopes for these cases were practically identical, as shown by the following tabulation:

Run 28	First Day	Second Day
d, depth, ft	0.285	0.284
U, mean velocity, ft/sec	1.31	1.32
S, slope	0.00245	0.00242

Other runs gave similar evidence of being very stable.

Experimental Results

Each run represented a stable uniform flow in equilibrium with its movable sand bed. Runs were established and the measurements were made according to procedures outlined in the previous section.

The sand was so fine that the rate of suspended load transport was much greater than the rate of bed load transport, except possibly in Runs 10 and 26 where the total transportation rate was very small. This does not mean that an insignificant amount of material was in motion on the bed; on the contrary, several layers of sand grains were almost always sliding along the bed, but at a relatively low rate compared with the velocity of the water. If, for example, the same amount of material was in suspension, it yielded a much larger transportation rate, (expressed as weight per unit time passing any cross section) because the sand then traveled at the full speed of the water. Anyway, the distinction between bed load and suspended load is quite nebulous, and no attempt was made here to make an artificial separation. All the values of transportation rate and discharge concentration reported in this section refer to the entire sediment discharge, bed load included.

For purposes of comparison, a few runs (C-1, C-2, C-3, C-4, C-6, C-7) were made without any sediment.

Summary of Data

In Table 1 are tabulated the most important measured and calculated quantities for each flow. In the following list of symbols, the items are in the order of their appearance in Table 1.

1. Q = discharge, in cubic feet per second.

2. d = average depth in feet in the reach of stable flow, determined from water surface and bed profiles.

3. $r = bd/(b + 2d)$ = hydraulic radius in feet. (b = width = 0.875 ft for all runs.)

4. S = slope of the energy line.

5. $U_* = \sqrt{grS}$ = shear velocity for the whole channel, feet per second.

6. $U = Q/bd$ = average velocity of flow for the entire channel.

7. $f = 8(U_* /U)^2$ = Darcy-Weisbach friction factor for the whole channel.

8. T = water temperature in degrees Centigrade.

9. r_b = hydraulic radius for the bed section in feet determined from the sidewall correction calculations. Since the sidewalls are hydrodynamically smooth, Johnson's basic procedure[6] (employing f and the Karman-Prandtl resistance equation for smooth pipes) was used, with a few minor modifications to make the calculations direct instead of by trial and error. The derivations and equations used, as well as a sample calculation are reported in detail in the author's thesis.[5]

10. $U_{*b} = \sqrt{gr_bS}$ = shear velocity for the bed section, feet per second.

11. $f_b = 8(U_{*b}/U)^2$ = friction factor for the bed alone, calculated from the sidewall correction procedure.

12. Number of discharge samples upon which $\bar{C}$ is based.

13. $\bar{C}$ = average sediment discharge concentration in grams per liter.

14. $G = 3.74\ \bar{C}\ Q$ = total sediment transportation rate (or discharge) in pounds per minute. (3.74 is the conversion factor for units.)

15. $F = U/\sqrt{gd}$ = Froude number.

16 and 17. Water surface condition and bed condition entries are included here because of their importance in the results. The terminology will be explained in the next section.

In the subheadings, D_s is the mean sedimentation diameter of the bed

6. "The Importance of Side-Wall Friction in Bed-Load Investigations," by J. W. Johnson, Civil Engineering, Vol. 12, No. 6, June 1942, pp. 329-331.

material. D_s for the sediment load is usually slightly less because of the sorting.

Where entries are missing in Table 1, either the item does not apply or the data were not obtained. Whenever there is a reasonable basis for making good estimates, when the data are missing, estimated values have been entered and designated by "e".

For all the entries in Table 1 only the significant figures are reported. The accuracy of the figures is thus indicated directly; for example, the discharge is reported to the nearest .005 cfs. Of all the runs, Runs 5, 8, 9, 10, and 11 are the least accurate, as shown by the entries for d, r, U_*, r_b, and U_{*b}, which are given only to the nearest 5 units in the third place. The second series, Runs 21 to 30, is more reliable because of slight improvements in experimental technique gained by experience with the earlier runs. However, there is no significant difference in the conclusions which may be drawn from each of the two series of runs.

Because of the limitations of the flow system, it was not possible to cover a wider range of conditions. Discharges less than 0.20 cfs were not used because sand storage in the return pipe could cause large errors in the discharge measurement and a shortage of sand in the flume section. Depths greater than 0.30 ft were not used because the effect of the walls becomes too large, and difficulty was encountered with the inlet condition. In addition, velocities giving Froude numbers in excess of 0.80 caused such high standing surface waves, that the flow was completely unmanageable. Nonetheless, in spite of these three restrictions, enough conditions were covered to lead to some valuable conclusions, which are believed to be qualitatively valid over a much wider range of conditions.

No attempt was made to derive an empirical formula for the transportation rate on the basis of these flume data. However, some qualitative relationships will be pointed out in "Discussion of Results."

Observations of Water Surface and Bed Configuration

When the mean velocity and the transportation rate were low, the dunes which formed were fairly large and haphazardly placed; the friction factors were as large as 0.13. As the velocity of flow was increased, the dune wavelength increased slightly, the pattern of the dunes became more regular, the dunes moved faster, and the friction factor decreased slightly. The height of the fully developed dunes did not change appreciably, possibly because of insufficient sand in the system to develop higher dunes.

The dune height in the central part of the channel was generally about 0.5 to 0.6 in. on the average and up to 1.5 in. maximum. The dune velocity ranged from $^1/5000$ to $^1/500$ of the mean stream velocity. The average wavelength varied from 4.3 to 5.6 in.

A typical dune configuration is shown in Fig. 5 for Run 30, which has a transportation rate intermediate between the highest and lowest of the runs with dunes.

As the velocity of flow was progressively increased, there always came a point where the sand tended to collect in a single long flat wave, which traveled perpetually through the system without changing its size or shape. The wave was a long thick deposit of sand with a smooth flat top, over which the water flowed with reduced depth, and increased velocity and sediment load. In the remainder of the flume, the bed would still be covered with rugged dunes for which the friction factor remained large. Pronounced sand waves were present in Runs 12 and 24, but since the flow was in equilibrium

both on the sand wave and away from it, the data in Table 1 pertain as noted either to the dune section or the sand wave separately. It is not apparent why these solitary sand waves were formed.

Meandering of the bed was another puzzling phenomenon that was observed for several runs with the 0.16 mm sand, although not with the smaller sand. The sand distributed itself non-uniformly across the channel by forming a ridge which weaved back and forth in the channel. The bed surface was covered with small sand ripples, and the bed friction factor was considerably less than for the runs with rugged dunes.

With a further increase in velocity (and transportation rate), the sand again spread itself uniformly throughout the flume, with the surface of the bed being quite smooth except for some small ripples near the wall, as shown in Fig. 4.

Up to Froude number F = 0.7 to 0.75, the water surface was relatively calm. At the lowest F = 0.27, the surface had a glassy smoothness, in spite of large bed irregularities. At higher Froude numbers before the dunes disappeared, the surface became quite rippled as a result of the disturbance to the flow by the bed. But when F reached some critical value between 0.7 and 0.75, large surface waves with wavelengths of 10 to 12 in. developed. A typical wave train would move slowly upstream, and finally disappear without breaking; new waves would form and repeat the process. In the presence of these surface waves the sand bed was always smooth in the center of the channel, except for very slight undulations of the same wavelength as the surface waves when the latter were unusually large.

At still higher Froude numbers, the waves get extremely high, sometimes as high as the average depth of flow, and long, low, rapidly-shifting antidunes are formed. No runs were performed at this extreme condition, because it was impossible to ascertain whether the flow was in equilibrium, and most of the usual measurements were impossible. It was also noted, however, that even at velocities substantially above critical (i.e., $F > 1$), the water surface never flattened out again as it would for clear water flow, but continued to be intensely wavy as long as there was a sand bed.

Discussion of Results

Transportation Relationships in the Laboratory Flume

The experimental data given in Table 1 have been plotted in several different ways in Figs. 6 to 11 in order to show how the total sediment discharge is related to the other characteristics of the laboratory streams. Since the bed load discharge is small, the total sediment discharge may be considered equivalent to the suspended load discharge for all practical purposes.

(1) Impossibility of Taking Slope or Shear as an Independent Variable. First of all, from Figs. 6 and 7 it appears that the mean velocity, U, and the sediment discharge concentration, $\bar{C}$, are not uniquely determined by the depth, d, and slope, S. For example, on Fig. 6 the points ($U = 2.04$, $\bar{C} = 1.95$) and ($U = 1.35$, $\bar{C} = 1.1$) have practically the same d and S values, but quite different velocity and concentration values. This result is completely contrary to present concepts of open channel hydraulics; it is commonly believed that if the depth, width and slope, and hence the shear, are known in addition to the size of material, that the velocity of flow and the sediment transportation rate can be calculated uniquely.

In applying any flow formula, such as the Manning equation or the Darcy-Weisbach formula to a flow over a movable bed of fine sand, the usual

assumption of approximately constant roughness leads to a gross misconception about the relationship of velocity to depth and slope. Actually, the "constant" roughness can change more than any of the other variables, because of the changing bed configuration. For example, increasing the slope of a stream with depth constant is supposed to increase its velocity, but from either Fig. 6 or 7 it is apparent that higher slopes are frequently associated with lower velocities for a given depth. Similarly, if the depth is increased, holding the slope constant, then the velocity should increase, whereas the experimental results show that this is not always the case.

Because the walls have a significant and variable effect on the flow, it is perhaps preferable to compare the data on the basis of the hydraulic radius for the bed, r_b, and the bed shear, obtained from the sidewall correction computations. (See Table 1.) When U and $\bar{C}$ are plotted again against S and r_b, instead of against S and d, and lines of constant shear (or simply constant r_bS) are drawn, the same conclusions may be drawn. If anything, one might even be led to conclude that the higher concentrations and velocities are associated with the smaller shears, a result completely contrary to widely held beliefs about sediment transportation.

In the range of experiments conducted, it may be noted from Table 1 that the shear velocity for the bed, U_{*b}, has remarkably little variation compared with the velocity or transportation rate. It may indicate that in the range of sediment loads encountered the bed configuration tends to adjust itself in a way which greatly reduces the variation of shear that might ordinarily be expected. This being the case, it is improbable that the suspended load transport of a stream can ever be expressed directly in terms of the shear.

(2) Depth and Velocity as Independent Variables. Since the velocity and the sediment discharge concentration cannot be expressed uniquely as a function of the slope and either the depth or the bed hydraulic radius, an attempt was made to find out which quantities could be logically used as independent variables. It was found that any two variables representing two of the following three groups could be considered as independent: (a) d or r_b; (b) U or Q; and (c) $\bar{C}$ or G. For Figs. 8 and 9, d and U were used as coordinates and for Figs. 10 and 11, Q and G. These two particular combinations of independent variables seemed most logical and made the results the easiest to interpret.

The sand size is, of course, a very significant independent variable, but since only two sizes of sand were used, little could be deduced about the effect of the sand size on the sediment load. No information was obtained on the behavior of a graded sand.

Although temperature is a variable of secondary importance, it was still found advisable to regulate it at 25°C for the second series of runs, Runs 21 and 30, to avoid any possible confusion resulting from too many variables changing at once. For the earlier runs, Runs 2 to 13, with 0.16 mm sand, the range in temperature was 21 - 27.5°C, except for three runs; these three cases are specially labeled on all the figures. When the temperature rises, the settling velocity of the sand increases, and there is a tendency for the sediment discharge to decrease, other things being equal (as may be seen from Table 1).

The following fairly definite qualitative conclusions may be drawn from Figs. 8 and 9 where U and d are taken as the independent variables, and two of the dependent variables, f_b and $\bar{C}$, are written in for each of the plotted points.

(a) f_b and $\bar{C}$ are uniquely determined by U and d, for the flume. From

these, all other quantities listed in Table 1 may be calculated, so that the character of the flow is completely determined by a selection of U and d. This was also borne out by general experience in operating the flume; by choosing the depth and mean velocity, one, and only one, definite equilibrium flow could be obtained.

(b) For a fixed depth, $\bar{C}$ increases as U increases in most cases. Also, f_b decreases markedly as U increases, because of the changes in the bed configuration. For example, in the case of the 0.16 mm sand, Fig. 8 shows that increasing the velocity from about 1.25 to 2.0 ft/sec for d = 0.195 ft, results in a drop in f_b from 0.102 to 0.0225 because the dunes are washed away, and the sand bed becomes flat. At the same time, the discharge concentration $\bar{C}$ is increased from 1.2 to 2.45 gr/1.

(c) For a fixed velocity, $\bar{C}$ decreases slightly as d increases. f_b does not appear to depend much on the depth in the limited range studied.

(d) The above conclusions apply equally well for both sand sizes.

(e) For a given d and U, the sediment discharge concentration $\bar{C}$ for the 0.10 mm sand, is roughly 2 to 4 times as much as for the 0.16 mm sand, in the range of conditions covered.

(f) f_b appears to be numerically the same for both sand sizes for any given d and U. Since f_b may be influenced by the amount of sediment load as well as the dune configuration, no further conclusions may be drawn here regarding the factors governing f_b.

In Fig. 9, it may be noted that there is a substantial gap between the points for runs with smooth bed and dune-covered bed. During the investigation an attempt was made to establish some points in the gap, but it appeared to be impossible. A sand wave would always form, thereby dividing the flume into a reach of higher velocity and smooth bed and another reach of lower velocity and rough bed. For the larger sand size (Fig. 8), three intermediate points were established, two at a velocity of about 1.8 ft/sec, and the other at 1.5 ft/sec; these were the three runs for which meanders were observed. However, for slightly lower velocities, the sand waves would probably interfere in the same way as for the 0.10 mm sand. It is not known whether this is a system effect, or whether there actually are impossible combinations of depth and velocity for these sand sizes.

(3) Water Discharge and Sediment Discharge as Independent Variables. Some corollaries of the conclusions above can be most easily deduced by replotting the data as in Figs. 10 and 11; the discharge, Q, and the total sediment transportation rate, G, are used as the independent variables, and the values of f_b and d are recorded opposite the plotted points. Note that the scales are logarithmic and that the length of the cycle in the Q scale is just twice that of the G scale. Additional conclusions based on Figs. 10 and 11 are as follows:

(a) f_b, d, and all the other variables can be uniquely determined from Q and G. For example, to transport 1.5 lb/min of 0.16 mm sand with a discharge of 0.25 cfs in the laboratory flume, it may be estimated from the neighboring points on Fig. 10 that the required depth will be about 0.19 ft and f_b = 0.07. For the same load of 0.10 mm sand, there follows from Fig. 11 that d = 0.27 ft and f_b = 0.12, with Q = 0.25 cfs, as before. The required slope in each case can be obtained by working back through the sidewall correction equations. It may be noted that in a long open circuit flume, Q and G are the only two variables which are independent and can be directly controlled. Everything else adjusts to these values.

(b) Maintaining a constant G, while Q is increased, requires that the

depth d increase, although not as much as Q. The velocity U also increases.

(c) For a constant Q, an increase in G requires a decrease in d. This is a significant conclusion, for it implies that the depth of flow depends on the sediment load for any given discharge; hence, a unique stage-discharge relation did not exist for the laboratory flume. For example, Run 23 and Run 28 have practically identical slopes and discharges (see Table 1), but two different depths, 0.189 ft and 0.284 ft respectively. The transportation rate for the former is 6.2 lb/min and for the latter 4.45 lb/min.

(d) For a given Q the largest f_b values are associated with the smallest sediment transportation rates, G. From Fig. 11 it appears that f_b reaches an upper limit of about 0.12 or 0.13 (very large values), and a further decrease in G will not increase f_b much more. Physically, there must be a limit to the roughness of the channel, because the height of the dunes is certainly limited by the depth of the water.

(e) The above conclusions are qualitatively the same for both sand sizes.

Although the data obtained are not sufficiently extensive to permit establishing general quantitative formulas, still the results shown in Figs. 10 and 11 can be represented schematically as shown in Fig. 12. The relationships indicated by the curves in this figure are very rough inasmuch as the figure is intended only to make some of the various qualitative conclusions more graphic.

Results of Other Laboratory Experiments

The multiplicity of the relation between sediment discharge and depth and slope encountered in the present experiments has not been pointed out by any previous laboratory investigators, as far as this author is able to ascertain. Either the roughness effect causing this multiplicity has not been noticed or else it has not been nearly so pronounced (if existent at all) for some of the other sand sizes heretofore used in sediment transportation experiments.

With coarse material, the sediment moves primarily as bed load; although dunes have been observed, it is possible that the roughness may not undergo nearly such radical changes. In this case, the bed load transportation rate would be more or less directly related to the shear, as first suggested by Du Boys, and checked by countless investigators during the past several decades. However, the large scatter of the points in all graphs of bed load transportation rates (which is usually ignored) is evidence that some other effect, such as roughness has not been taken full account of. In some bed load formulas, notably that of the Waterways Experiment Station (Vicksburg), the Manning roughness coefficient n has been included in the formula as an independent variable; however, a method for predicting n is then needed. Einsteins's bed load theory[7] takes account of variable channel roughness and includes a method for determining it. His method will be discussed later in connection with suspended load theories.

It is doubtful whether changes in roughness play a very significant part in the mechanics of streams with beds of silt or clay, or the finely ground silica flour used by Kalinske and Hsia.[8] It seems that dunes may never become

7. "The Bed-Load Function for Sediment Transportation in Open Channels," by H. A. Einstein, U.S. Dept. of Agr., (Soil Conservation Service), Tech. Bull. No. 1026, September 1950.
8. "Study of Transportation of Fine Sediments by Flowing Water," by A. A. Kalinske and C. H. Hsia, Univ. of Iowa Studies in Engrg. Bull. No. 29, 1945.

very large in this material because it is so easily moved by flowing water even at very low velocities. For nine different flows over a bed covered with this extremely fine material, Hsia found that the Manning n varied only from .0119 to .0098, with one of the highest values being for the run with the smallest sediment discharge concentration (0.64%) and the smallest n for the run with the largest concentration (11.1%). He found that the transportation rate did have a direct relation to the bed shear, increasing when the shear increased.

Consequently, it may be that the multiple relationship between sediment transportation rate and bed shear occurs only for some intermediate range of sediment sizes under ordinary circumstances; or possibly it would occur only at unusually large shears for coarse material, and at extremely small shears for fine materials.

Field Observations of Natural Streams

Some recent observations by the Missouri River Division of the Corps of Engineers[9] showed how rugged and irregular the bed of a river can be. In a sediment transportation study reach on the Missouri River near Omaha, periodic soundings of the sandy bottom have shown that the character of the bed surface is quite changeable, being smooth at some times or places and rugged and irregular (much like the bed of the flume) at other times or places. A number of examples show that the elevation of the bottom of the stream changes by as much as six or seven feet in distances of less than 50 feet in a flow 10 to 15 feet deep on the average.

Although it was not possible to make a thorough analysis of large river dunes, the evidence indicates that the size of dunes in fine sand is more or less proportional to the depth of flow. Since the dune size is relative, the Darcy-Weisbach f, which is a function of the relative roughness, should be used to characterize a dune configuration instead of the Manning n, which is directly related to the actual size of the roughness elements by Strickler's formula. If the channel roughness for the laboratory runs had been reported as the Manning n, the values obtained would seem small compared with values obtained for natural rivers; a bed surface covered with dunes 0.5 in. high in a flow 3 in. deep is very rough, whereas in a flow several feet deep, the same absolute bed configuration would be considered relatively smooth.

For large natural rivers, Leopold and Maddock[10,11] have found that the roughness, and hence the depth and velocity of flow are greatly affected by the amount of sediment load the stream must carry. Their observations agree well with the flume data from which the same conclusion was deduced.

For example, Leopold and Maddock in their Fig. 28[10] give the following information. When Hoover Dam was built on the Colorado River, the sediment load below Hoover Dam was sharply reduced because of the desilting action of the reservoir. At Yuma, Arizona, the annual suspended sediment

9. "Sediment Transportation Characteristics Study," by Missouri River Div., U.S. Corps of Engineers, First Interim Report, Appendix D, Topographic Maps, Omaha, Neb., Feb. 1952.
10. "The Hydraulic Geometry of Stream Channels and Some Physiographic Implications," by L.B. Leopold and T. Maddock, Jr., U.S. Geological Survey Professional Paper 252, 1953.
11. "Relation of Suspended-Sediment Concentration to Channel Scour and Fill," by L.B. Leopold and T. Maddock, Jr., Proc., Fifth Hydraulics Conf., State Univ. of Iowa Studies in Eng. Bull. 34, 1952, pp. 159-178.

discharge after the dam closure was only one tenth of what it had been previously. For a typical flow of 12,000 cfs, the depth jumped from about 5 to 9 feet, the velocity decreased from about 5 ft/sec to 3 ft/sec, and the Manning n increased from about .013 to .030 (which corresponds to a five-fold increase in the Darcy-Weisbach f). The slope did not change, although the steam bed elevation dropped about 8 feet in the first 10 years after construction of the dam. Here is a case where for a given discharge and slope, the stream responded quite differently when the suspended load was artificially reduced. Qualitatively the changes which have occurred are in perfect agreement with the conclusions based on the laboratory data plotted in Figs. 10 and 11.

Leopold and Maddock suggest that the increase in roughness was an effect of the decrease in suspended load. However, it is doubtful if the effect is direct; probably the bed of the stream has become covered with irregular dunes giving increased resistance to the flow. Actually, it is fortunate that the stream can adjust its bed, change the hydraulics of the stream, and thus greatly reduce its transporting capacity; for otherwise, the degradation at Yuma would undoubtedly have been much more severe.

Another example is given by Leopold and Maddock in their Fig. 23[10] (or Fig. 4[11]) showing the changes in the hydraulic characteristics of the Colorado River at Grand Canyon, Arizona, occurring during the annual flood of December, 1940, to June, 1941. On the rising stage, the suspended load was very large, and the stream bed aggraded; on the falling stage the load was much less, and the bed was scoured. For two approximately equal discharges, the following data may be read from two sets of points on their curves:

	Rising Stage	Falling Stage
Discharge, cfs	13,000	12,000
Suspended load discharge, tons per day	500,000	47,000
Depth, ft	9.5	16.5
Velocity, ft/sec	5	2.6
Gage height, ft	8	7.5
Stream bed elevation, ft	-1	-9
Width, ft	290	290

Presumably the slope of the river was approximately the same in both cases, as the stage was not changing rapidly. Assuming a constant slope, then, it may readily be shown that the Darcy-Weisbach friction factor f is 6.5 times as large on the falling stage as on the rising stage; and on the falling stage, when the transportation rate is small, the bed shear is actually much larger.

A further examination of Leopold and Maddock's Fig. 23[10] reveals that for a given depth, the discharge depends on the amount of load. For instance, when the depth is 16.5 ft, the discharge on the falling stage is 12,000 cfs, as indicated in the tabulation above, whereas it is 40,000 cfs, by interpolation, on the rising stage, when the sediment discharge is large.

In this second example also, the observed changes in the Colorado River during the passage of a flood agree qualitatively with conclusions drawn from the flume experiments. The Colorado River is an especially good example for illustrating these effects because the sediment load, carried mostly in suspension, is large and variable, and the bed is predominantly fine sand.

On another point, however, there is an apparent contradiction between Leopold and Maddock's observations and the flume data. From a massive collection of data on natural rivers, they conclude (p. 24[10]) that "a wide river having a particular velocity is observed to carry a smaller suspended load

than a narrow river having the same velocity and discharge." In other words, for a fixed velocity, the suspended load discharge concentration increases when the depth is increased. This is just the opposite of what has been concluded from the flume experiments where the sand size is held fixed.

However, their statements applies to streams as found in their natural condition, it does not mean then that if any given stream were artificially made narrower and deeper that the sediment discharge would increase. Since Leopold and Maddock compare a number of different streams, the size of the bed material is a variable which is not taken into account. Perhaps it might be construed on the basis of both Leopold and Maddock's conclusions and the results of the flume experiments that the narrow streams carry more load only because they are typically cut in finer material than the wider streams; but when the bed material is the same, there is flume evidence to show that a wide shallow stream would carry more suspended load at a given velocity.

When the sediment size is essentially fixed, the writer's conclusion regarding the effect of depth on concentration is supported by field observations of Hill[12] published in 1926, for the Colorado River at Yuma. A curve given by him shows that as the depth increases, the average concentration decreases with the velocity remaining constant.

Evaluation of Theories for Suspended Load Discharge

From the foregoing discussion of both flume and river data, it is evident that any workable suspended sediment transportation theory must include a way in which to determine the stream roughness, for it is not adequate to assume that it is constant. In fact the use of the ordinary fixed-bed open channel hydraulics for alluvial streams can lead to gross errors, since in any typical flow equation the roughness can change as much if not more than, all the other variables.

The bed pickup theory advanced by Lane and Kalinske[13] is simplified by consideration of the pickup mechanism from a flat bed surface only, whereas it has been observed in the flume that the existence of dunes greatly facilitates the suspension of bed material. Thus, a more general theory is needed which takes account of the changing bed configuration and its tremendous effect on the pickup mechanism. Furthermore, the original anlaysis of Lane and Kalinske and subsequent modifications thereof[8] were based on the common supposition that the shear could be used as an independent variable.

Einstein and Barbarossa[14] have made a significant attempt to analyze river channel roughness. A basic implication of their approach also is that there is a definite stage-discharge relationship for a stream, and that there will be a particular roughness associated with each point on the rating curve. This is equivalent to assuming that there is a unique relation between discharge, and depth and slope, irrespective of the sediment load. There is now strong evidence both from the field and the laboratory, indicating that this is not a basically correct physical law. It may appear true for some natural streams, inasmuch as it is not easy to isolate the effects of the various

12. Discussion by R. A. Hill of "Permissible Canal Velocities," by S. Fortier and F. C. Scobey, Trans., ASCE, Vol. 89 (1926) pp. 961-964.
13. "The Relation of Suspended to Bed Material in Rivers," by E. W. Lane and A. A. Kalinske, Trans., Am. Geophys. Un., Part IV, Vol. 20 (1939), pp. 637-641.
14. "River Channel Roughness," by H. A. Einstein and N. L. Barbarossa, Trans., ASCE, Vol. 117 (1952) pp. 1121-1146.

variables until something unusual happens to the stream, such as being dammed.

The analysis of Einstein and Barbarossa appears consistent because their basic curve for finding the channel roughness is based on stream data calculated from a stage-discharge curve for each stream. Consequently, their analysis will probably be found adequate for applying to the natural streams from which it was derived, but of limited help in predicting what changes would occur in the stream whose equilibrium is drastically upset by man-made works.

Einstein's basic transportation theory[7] is based on this analysis of roughness. An examination of his theory will reveal that for a given bed material, there is only one equilibrium rate of flow and sediment transportation rate corresponding to each combination of bed hydraulic radius and slope. Experiments performed by this writer show that this is certainly not true for fine sand in the laboratory flume, and it is the opinion of the writer, with some substantiating evidence from Leopold and Maddock[10,11] that this assumption is not always correct for natural rivers either. Therefore, it is believed that Einstein's basic approach to the determination of channel friction and suspended load discharge will have to be modified.

Blench[15] in his regime theory for canals has avoided consideration of basic quantities such as the shear. Since his analysis is purely empirical, it may be that he has arrived at some results which are just as reasonable as those of other investigators, who have assumed that the transportation rate of suspended sediment depended on the shear. However, it is difficult to evaluate his work because he has not reported any of the basic data, but only some of the constants in various empirical equations.

In view of the complexity of suspended load phenomena, it is no wonder that there has been considerable difficulty in finding a reasonable theory for determination of the suspended load transporting capacity of streams flowing over movable beds. This author has no basic theory to propose, and has sought only to establish some basic facts with which to evaluate existing theory. At present, there is great need for more good information on exactly what happens in such streams; without it sediment transportation theory will remain a jumble of approximate theories based on a great variety of different assumptions.

SUMMARY OF CONCLUSIONS

The principal conclusions may be summarized as follows:

(1) For the laboratory flume it was found that neither the velocity nor the sediment discharge concentration can be expressed as a single-valued function of the bed shear stress, or any combination of depth and slope, or bed hydraulic radius and slope. This finding is contrary to the commonly held assumption that the suspended sediment transporting capacity of a stream can be uniquely related to the geometry of the stream cross section, the slope of the channel, and the size of the bed material. Two present theories, based on this erroneous assumption, were found to be inadequate.

(2) The difficulty in relating the velocity and concentration to the bed shear arises because the changeable bed configuration causes extremely

15. "Regime Theory for Self-Formed Sediment-Bearing Channels," by T. Blench, Trans., ASCE, Vol. 117 (1952), pp. 383-408.

large variations in the channel roughness. For the 0.10 mm sand the Darcy-Weisbach friction factor for the bed varied from 0.019 for a run for which the velocity was high and the sand bed was swept smooth, to 0.13 for a run for which the velocity was low and the bed became covered with a stable pattern of large irregular dunes.

(3) When either mean velocity and depth, or water discharge and total sediment discharge are used as the pair of independent variables for the flume data, all other quantities are uniquely determined by what appears to be an orderly and logical relationship among all the various variables.

(4) From the data obtained in the laboratory flume with 0.10 mm and 0.16 mm sand, the following qualitative relationships were found.

(a) For constant discharge Q, an increase in the sediment discharge G requires a decrease in the depth, d. Field observations indicate that this relationship is also true for some large rivers.

(b) For a given slope and discharge, it was found that two depths of flow were possible. When the sediment discharge is small, the depth is large, the velocity is small, and the bed is rough; and when the sediment discharge is large, the depth is small and the bed of the laboratory channel is smooth. Field data show that this conclusion applies to natural streams as well.

(c) If the water discharge Q is to be increased without changing the total sediment discharge G, then an increase in d is necessary, although this increase is relatively less than in Q.

(d) For a given Q, the largest bed friction factors are associated with the lowest values of G.

(e) When the mean velocity of the stream U increases, with no change in the depth, d, the bed friction factor f_b decreases, and the sediment discharge concentration $\bar{C}$ generally increases.

(f) When d is increased, holding U constant, $\bar{C}$ decreases slightly, and f_b does not change appreciably in the range of conditions covered. Field evidence supports this conclusion.

(g) For a given d and U, f_b seems to be numerically the same for both sand sizes used. The bed configurations also appeared the same.

(h) For a given d and U, the discharge concentration $\bar{C}$ for the 0.10 mm sand was 2 to 4 times as large as $\bar{C}$ for the 0.16 mm sand.

A list of the symbols used may be found under "Experimental Results, Summary of Data."

ACKNOWLEDGMENTS

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During the academic year, 1952-53, the writer carried on the research with the aid of a National Science Foundation Predoctoral Fellowship.

TABLE 1

SUMMARY OF EXPERIMENTAL RESULTS

Run No.	Q Dis-charge cfs	d Depth ft	r Hydr. Radius ft	S Slope	U_* Shear Vel. ft/sec	U Ave. Vel. ft/sec	f Frict. Factor	T Water Temp. ^oC	r_b Bed Hydr. Radius ft	U_{*b} Bed Shear Vel. ft/sec	f_b Bed Frict. Factor	No. of Sed. Disch. Samples	$\overline{C}$ Sed. Disch. Conc. gr/l	G Sed. Dis-charge lb/min	F Froude No.	Water Surface Condition	Bed Condition	Run No.
								Clear Water										
C-1	0.695	0.232	0.152	0.0050	0.157	3.44	0.0167	18	0.156	0.159	0.017	-	-	-	1.27	Smooth	No sand	C-1
C-2	0.45e	0.171	0.123	0.0050	0.141	3.0e	-	18.5	-	-	0.0185e	-	-	-	1.3e	Smooth	No sand	C-2
C-3	0.24e	0.115	0.091	0.0050	0.121	2.4e	-	18	-	-	0.0205e	-	-	-	1.3e	Smooth	No sand	C-3
C-4	0.32e	0.225	0.148	0.00125	0.077	1.6e	-	14.5	-	-	0.0195e	-	-	-	0.6e	Smooth	No sand	C-4
C-6	0.47	0.179	0.127	0.0049	0.142	3.00	0.0180	27	0.131	0.145	0.0185	-	-	-	1.25	Smooth	No sand	C-6
C-7	0.435	0.241	0.155	0.00195	0.099	2.06	0.0183	17	0.157	0.099	0.0185	-	-	-	0.74	Small waves	No sand	C-7
								Sand Bed, D_s = 0.16 mm										
2	0.54e	0.284	0.172	0.0018	0.100	2.15e	0.017e	17	0.174e	0.100	0.018e	0	-	-	0.71e	Small waves	Smooth	2
3	0.435	0.243	0.156	0.0025	0.112	2.04	0.024	22	0.178	0.120	0.0275	5*	1.95*	3.1	0.73	Waves	Smooth	3
4	0.43	0.236	0.153	0.0024	0.108	2.08	0.022	12.5	0.164	0.112	0.023	5	2.45	3.9	0.75	Waves	Smooth	4
5	0.28	0.18	0.13	0.0031	0.115	1.8	0.033	26	0.145	0.12	0.038	5	1.9	2.0	0.74	Lge ripples	Meanders	5
6	0.345	0.195	0.135	0.0024	0.103	2.00	0.021	21	0.143	0.106	0.0225	6	2.45	3.1	0.80	Lge waves	Smooth	6
7	0.435	0.243	0.156	0.0021	0.103	2.04	0.020	31.5	0.170	0.107	0.0225	7**	2.15**	3.5	0.73	Waves	Smooth	7
8	0.375	0.24	0.155	0.0023	0.11	1.75	0.030	27.5	0.185	0.12	0.036	6	1.5	2.1	0.63	Ripples	Meanders	8
9	0.285	0.245	0.155	0.0026	0.115	1.35	0.059	27.5	0.21	0.13	0.079	12	1.1	1.2	0.47	Ripples	Dunes	9
10	0.20	0.25	0.16	0.0020	0.10	0.93	0.095	24	0.225	0.12	0.135	4	0.2	0.15	0.33	Smooth	Dunes	10
11	0.205	0.155	0.115	0.0033	0.11	1.5	0.043	26	0.135	0.12	0.050	6	2.7	2.1	0.67	Lge ripples	Meanders	11
12##	0.37	0.30	0.178	0.0022	0.111	1.40	0.050	26	0.248	0.131	0.070	6	0.72	1.0	0.45	Ripples	Dunes	12##
13	0.215	0.197	0.136	0.0035	0.124	1.25	0.078	26.5	0.178	0.142	0.102	15	1.2	0.95	0.50	Lge ripples	Dunes	13
								Sand Bed, D_s = 0.10 mm										
21	0.435	0.236	0.154	0.00225	0.106	2.10	0.0205	25.0	0.166	0.110	0.022	6	4.85	7.9	0.76	Waves	Smooth	21
21a	0.435	0.236	0.154	0.0022	0.104	2.10	0.020	25.0	0.165	0.108	0.0215	6	4.9	8.0	0.76	Lge waves	Smooth	21a
23	0.325	0.189	0.132	0.00245	0.102	1.96	0.0215	25.0	0.141	0.106	0.023	12	5.1	6.2	0.79	Lge waves	Smooth	23
24#	0.265	0.226	0.149	0.0028	0.116	1.34	0.060	25.0	0.197	0.133	0.079	6	4	4	0.50	Ripples	Dunes	24#
24#	0.265	0.17	0.12	-	-	1.8	-	25.0	-	-	-	6	7	7	0.76	Small waves	Low ripples	24#
25	0.20	0.187	0.131	0.0033	0.118	1.23	0.074	25.0	0.168	0.134	0.095	10	5.3	4.0	0.50	Lge ripples	Dunes	25
26	0.20	0.279	0.170	0.0013	0.084	0.82	0.084	25.0	0.245	0.101	0.12	6	0.19	0.14	0.27	Smooth	Dunes	26
27	0.20	0.231	0.151	0.00235	0.107	0.99	0.094	25.0	0.207	0.126	0.13	11	1.35	1.0	0.36	Small ripples	Dunes	27
28	0.33	0.284	0.172	0.0024	0.116	1.32	0.062	25.0	0.244	0.138	0.088	12	3.6	4.45	0.44	Ripples	Dunes	28
29	0.52	0.280	0.171	0.00185	0.101	2.13	0.0180	25.2	0.178	0.103	0.019	15	3.45	6.7	0.71	Smooth	Smooth	29
30	0.265	0.281	0.171	0.00215	0.108	1.08	0.080	25.0	0.246	0.130	0.115	12	1.75	1.7	0.36	Small ripples	Dunes	30

e - estimated

* Also 5 samples at 17.5°C, $\overline{C}$ = 2.1 gr/l; and 5 at 12.5°C, $\overline{C}$ = 2.35 gr/l

** Also 7 samples at 28°C, $\overline{C}$ = 1.8 gr/l; and 10 at 26°C, $\overline{C}$ = 1.6 gr/l

\# Dual equilibrium due to a long flat sand wave. See text.

\## Long flat sand wave present, but data pertain only to rugged dune section.

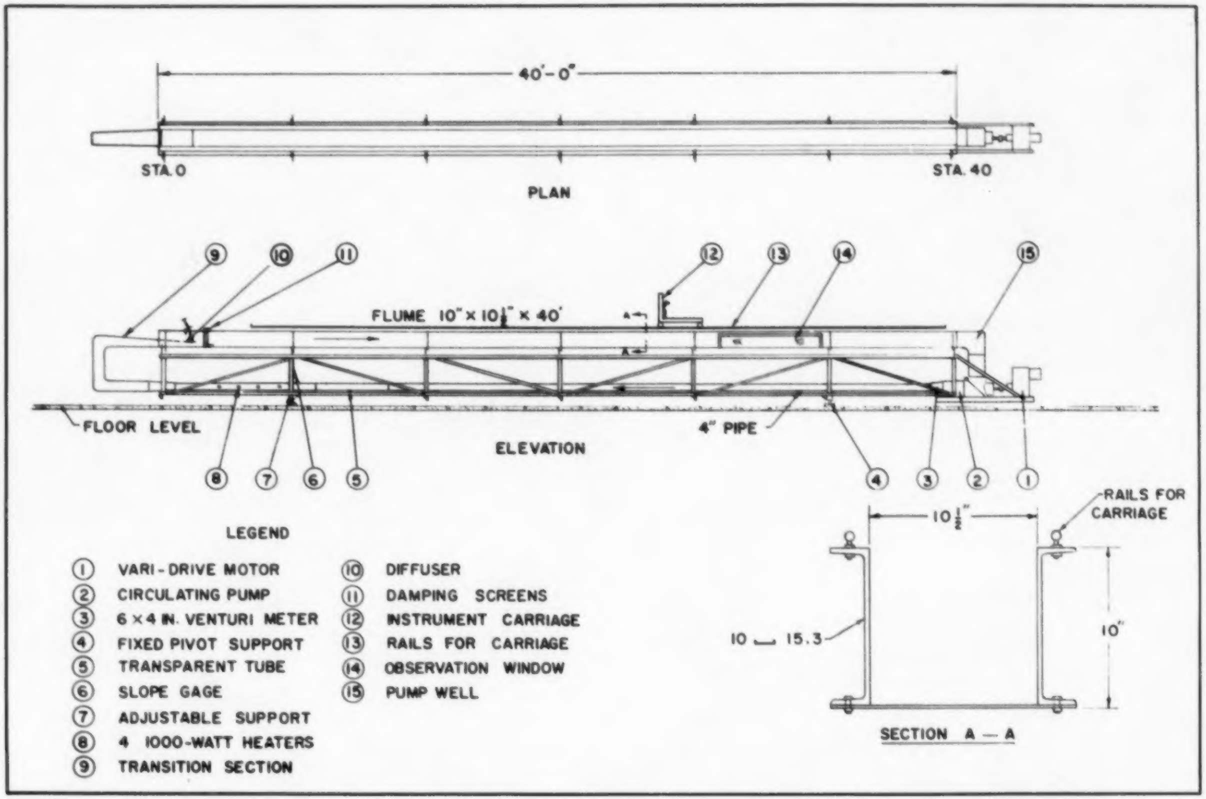


Fig. 1. Schematic diagram of the flume.

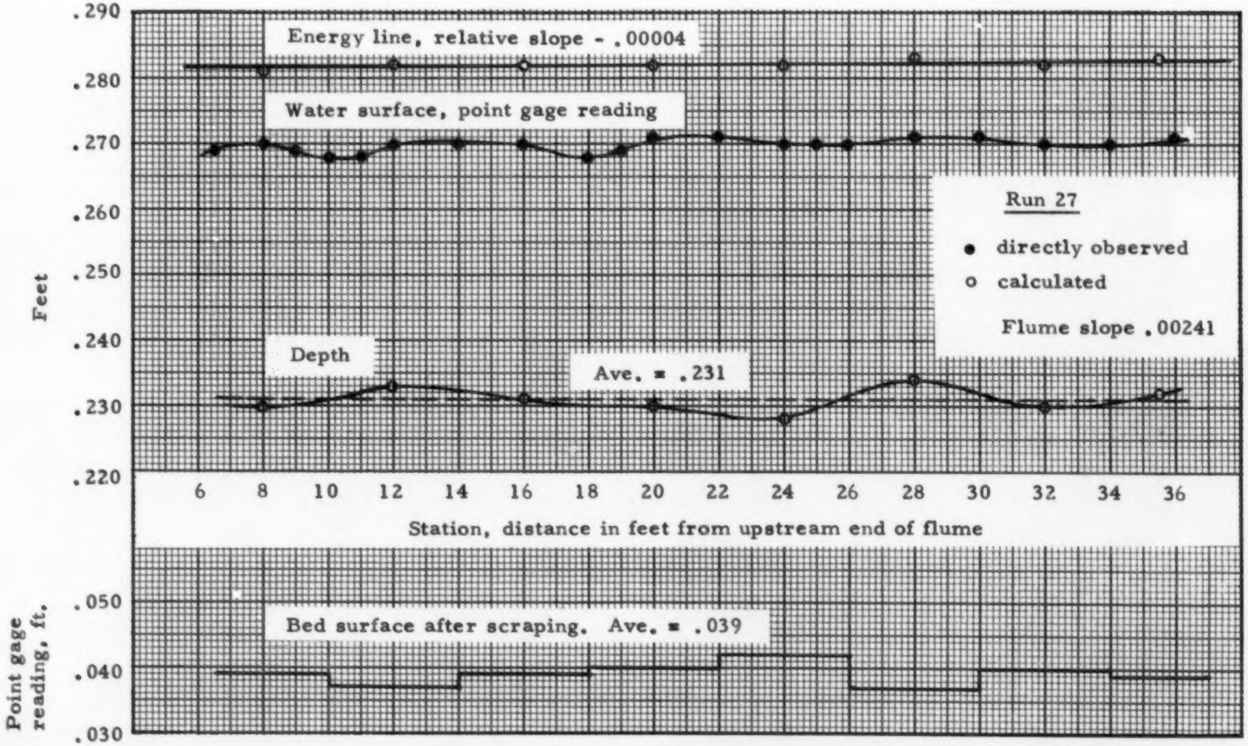


Fig. 2. Typical flume profiles for a run with dunes on the bed and small ripples on the water surface

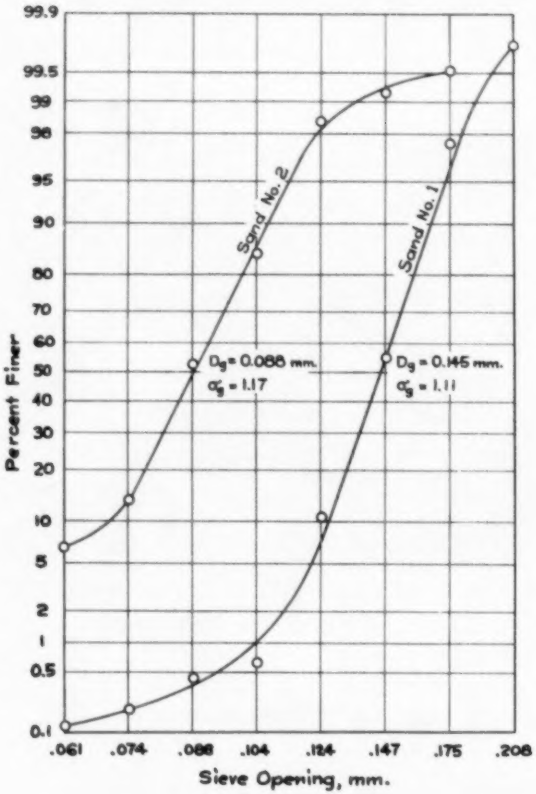


Fig. 3. Sieve analyses of sands used.

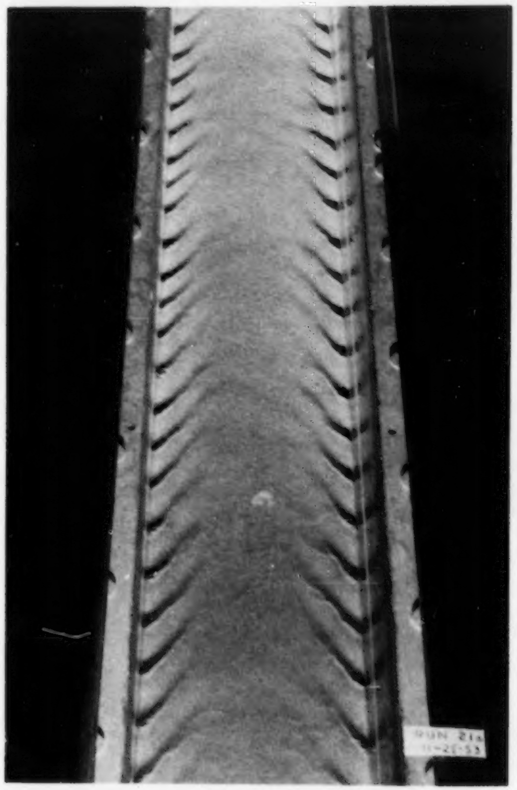


Fig. 4. Typical overhead view, for runs with smooth bed, looking downstream. Bed friction factor, $f_b = 0.0215$ (Run 21a).

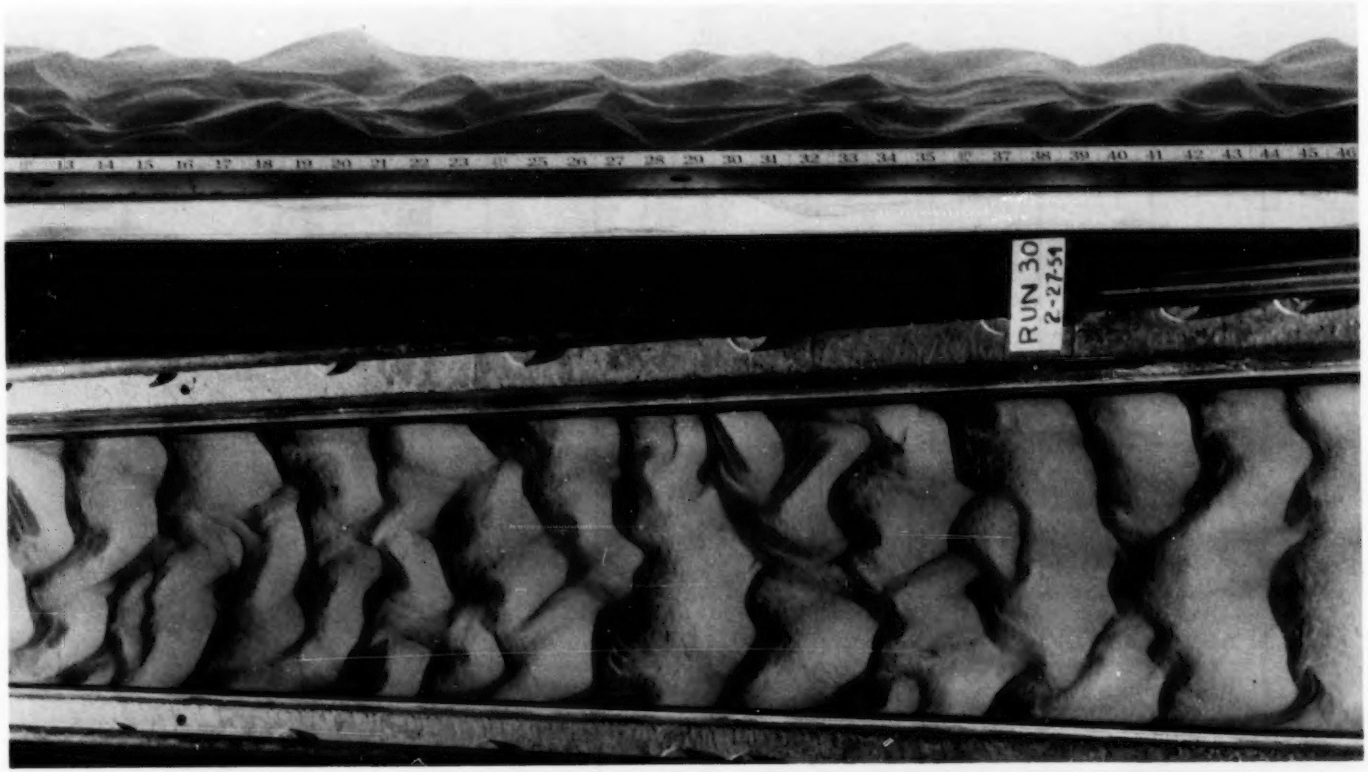
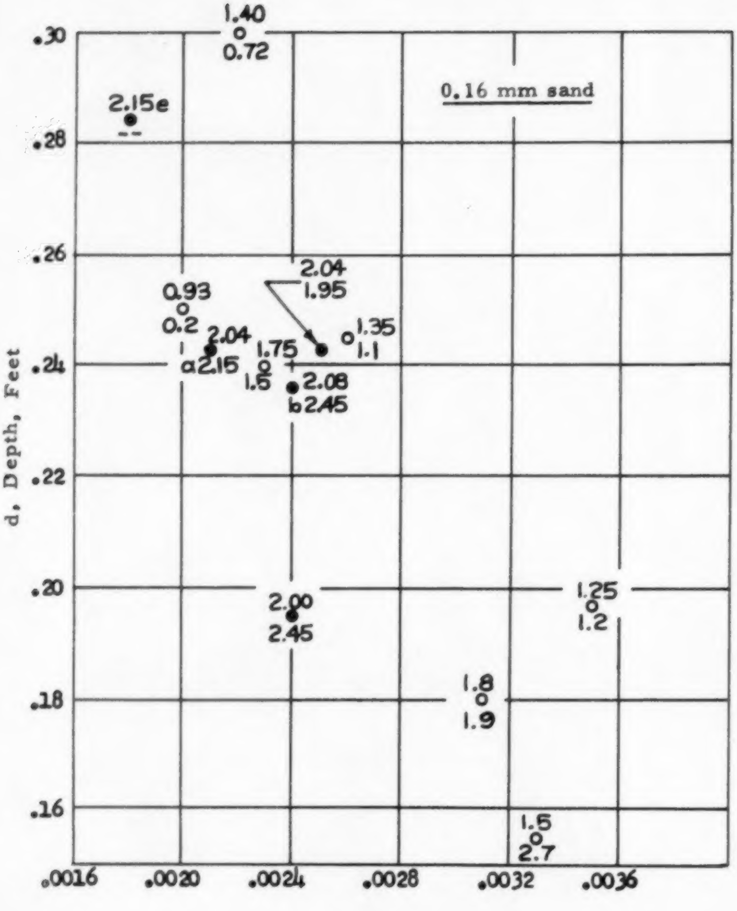


Fig. 5. Side and top views of a typical dune configuration. Downstream direction is to the right. Bed friction factor, $f_b = 0.115$ (Run 30).

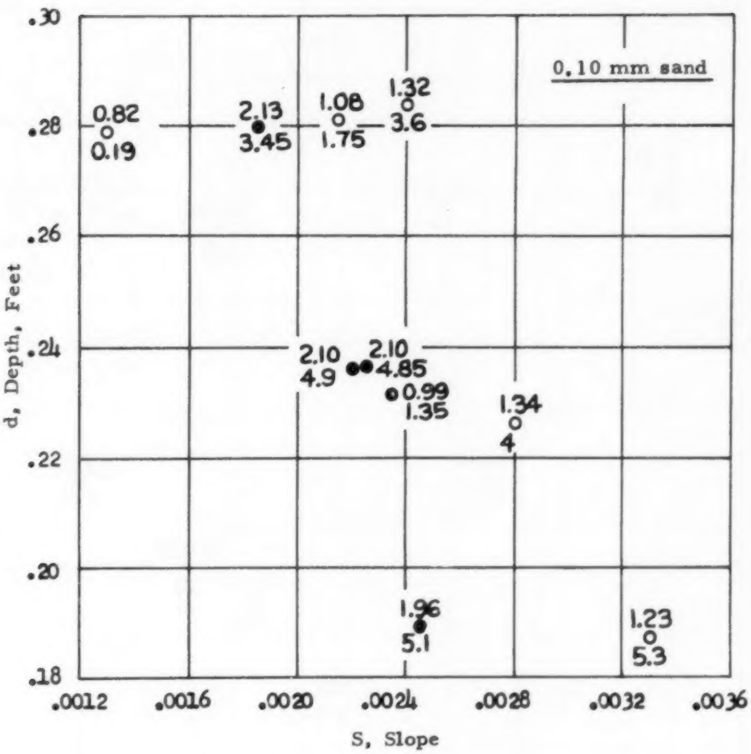


Legend: ● Bed smooth   o Bed covered with dunes

Upper figure: U = mean velocity, ft/sec
Lower figure: $\bar{C}$ = sed. disch. concentration, gr/l

a: T = 31.5°C.
b: T = 12.5°C.
e: estimated and T = 17°C.

Fig. 6. Velocity and concentration vs. depth and slope for 0.16 mm sand

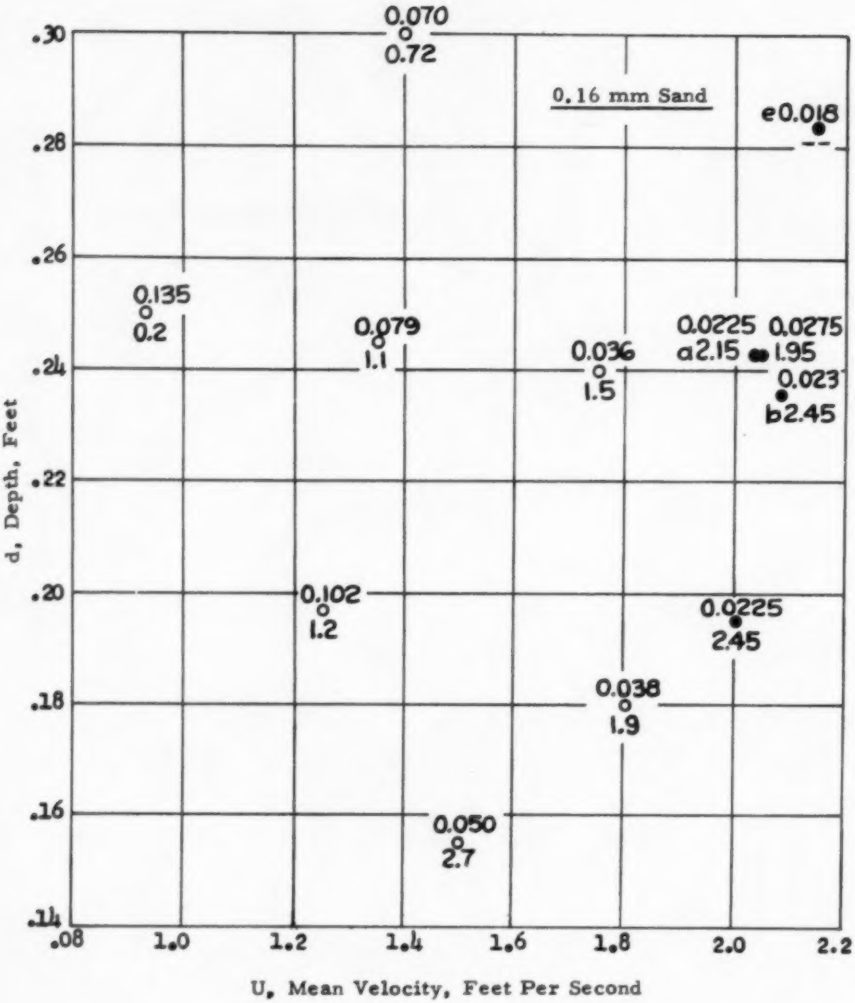


Legend: ● Bed smooth

O Bed covered with dunes

Upper figure: U = mean velocity, ft/sec
Lower figure: $\bar{C}$ = sed. disch. concentration, gr/l

Fig. 7. Velocity and concentration vs. depth and slope for 0.10 mm sand

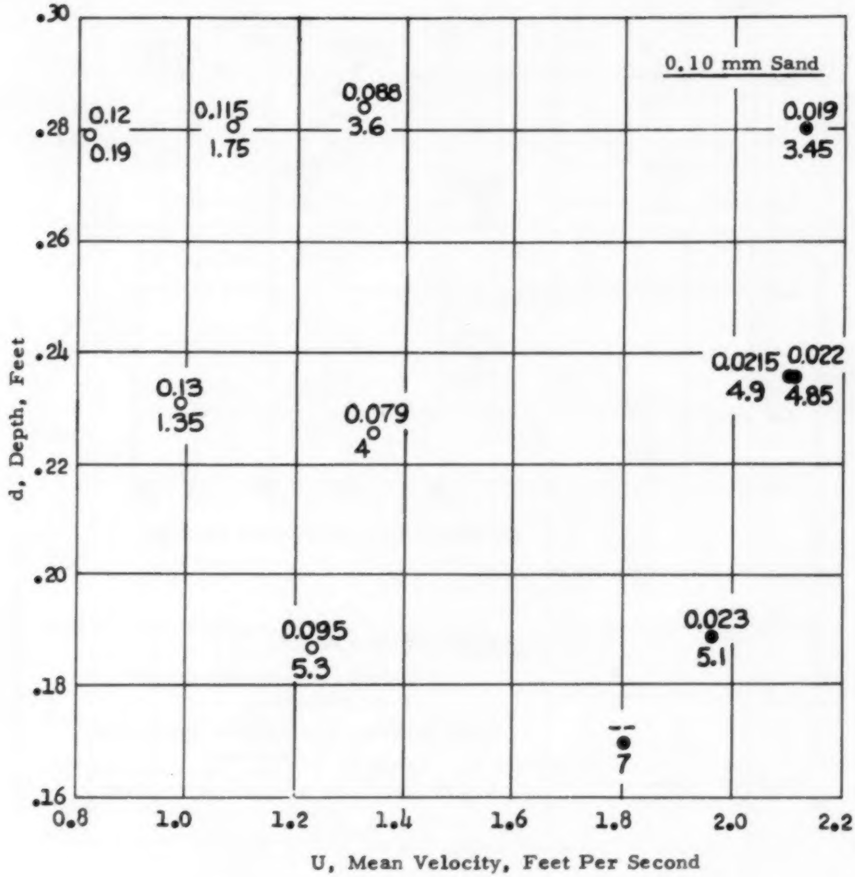


Legend: ● Bed smooth O Bed covered with dunes or meanders

Upper figure: f_b = friction factor for the bed
Lower figure: $\overline{C}$ = sed. disch. concentration, gr/l

a: T = 31.5°C.
b: T = 12.5°C.
e: estimated and T = 17°C.

Fig. 8. Bed friction factor and concentration vs. depth and velocity for 0.16 mm sand



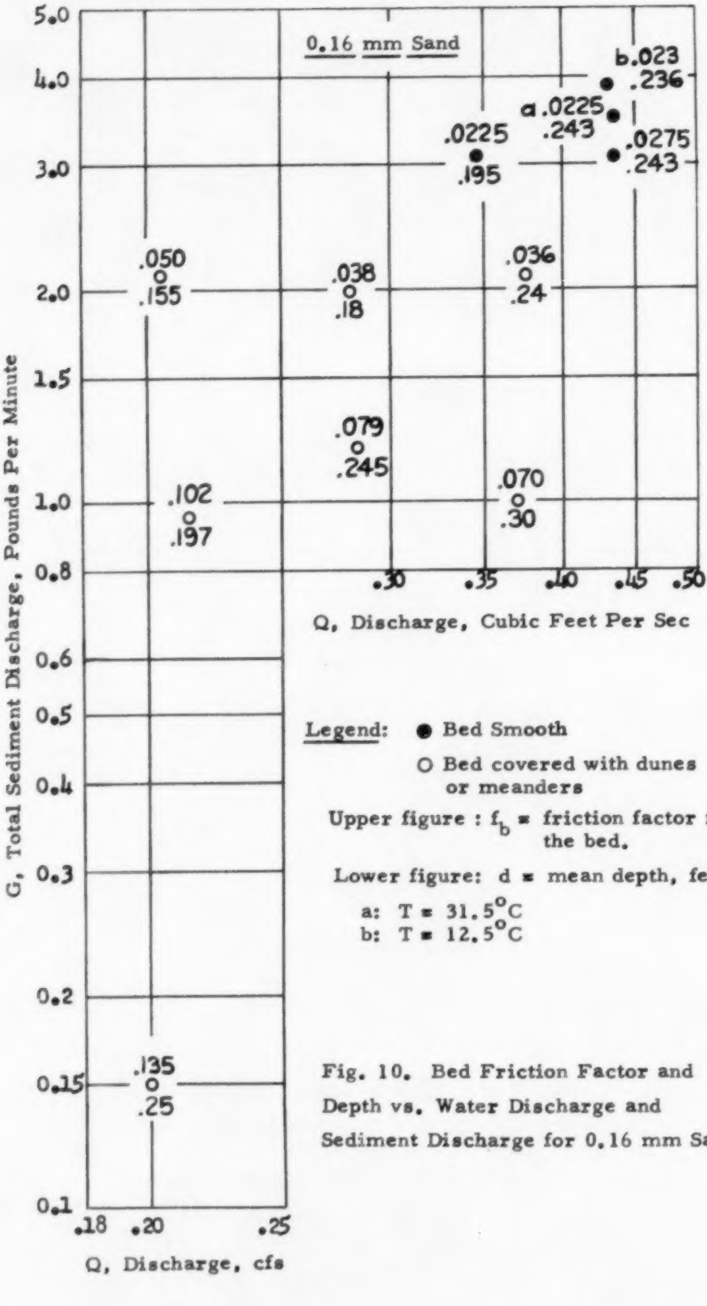
Legend: ● Bed smooth

O Bed covered with dunes

Upper figure: f_b = friction factor for the bed

Lower figure: $\overline{C}$ = sed. disch. concentration, gr/l

Fig. 9. Bed friction factor and concentration vs. depth and velocity for 0.10 mm sand



Legend: ● Bed Smooth

○ Bed covered with dunes or meanders

Upper figure : f_b = friction factor for the bed.

Lower figure: d = mean depth, feet

a: T = 31.5°C
b: T = 12.5°C

Fig. 10. Bed Friction Factor and Depth vs. Water Discharge and Sediment Discharge for 0.16 mm Sand.

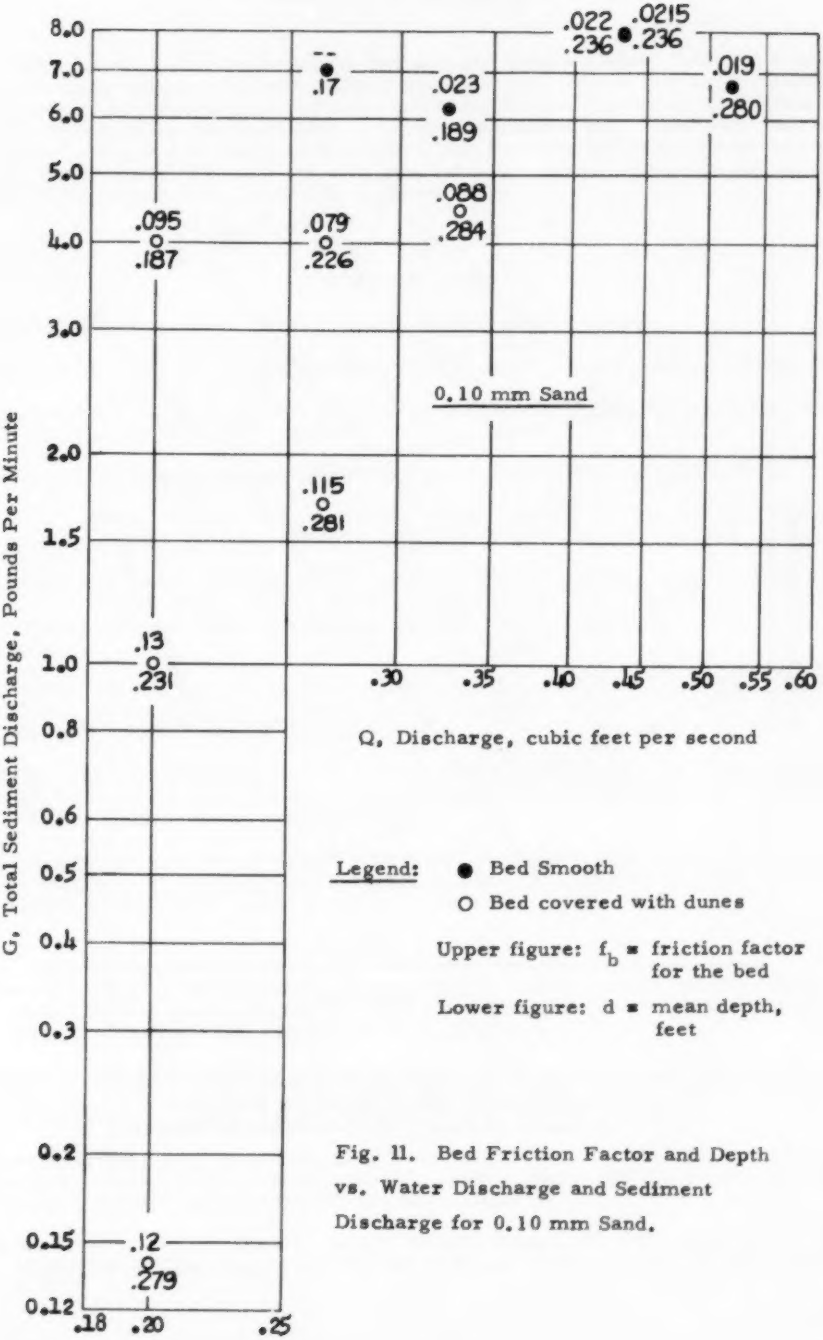


Fig. 11. Bed Friction Factor and Depth vs. Water Discharge and Sediment Discharge for 0.10 mm Sand.

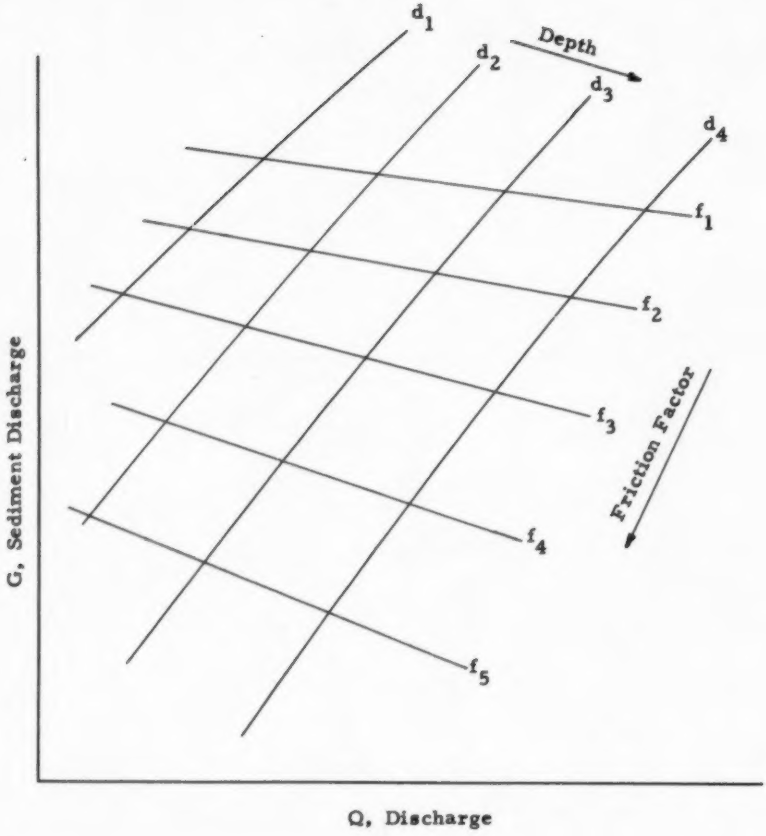


Fig. 12. Schematic representation of suspended sediment transportation relationships for the flume. (Increase of d and f is in direction of arrows.)

PROCEEDINGS-SEPARATES

The technical papers published in the past year are presented below. Technical-division sponsorship is indicated by an abbreviation at the end of each Separate Number, the symbols referring to: Air Transport (AT), City Planning (CP), Construction (CO), Engineering Mechanics (EM), Highway (HW), Hydraulics (HY), Irrigation and Drainage (IR), Power (PO), Sanitary Engineering (SA), Soil Mechanics and Foundations (SM), Structural (ST), Surveying and Mapping (SU), and Waterways (WW) divisions. For titles and order coupons, refer to the appropriate issue of "Civil Engineering" or write for a cumulative price list.

VOLUME 80 (1954)

APRIL: 428(HY)[c], 429(EM)[c], 430(ST), 431(HY), 432(HY), 433(HY), 434(ST).

MAY: 435(SM), 436(CP)[c], 437(HY)[c], 438(HY), 439(HY), 440(ST), 441(ST), 442(SA), 443(SA).

JUNE: 444(SM)[e], 445(SM)[e], 446(ST)[e], 447(ST)[e], 448(ST)[e], 449(ST)[e], 450(ST)[e], 451(ST)[e], 452(SA)[e], 453(SA)[e], 454(SA)[e], 455(SA)[e], 456(SM)[e].

JULY: 457(AT), 458(AT), 459(AT)[c], 460(IR), 461(IR), 462(IR), 463(IR)[c], 464(PO), 465(PO)[c].

AUGUST: 466(HY), 467(HY), 468(ST), 469(ST), 470(ST), 471(SA), 472(SA), 473(SA), 474(SA), 475(SM), 476(SM), 477(SM), 478(SM)[c], 479(HY)[c], 480(ST)[c], 481(SA)[c], 482(HY), 483(HY).

SEPTEMBER: 484(ST), 485(ST), 486(ST), 487(CP)[c], 488(ST)[c], 489(HY), 490(HY), 491(HY)[c], 492(SA), 493(SA), 494(SA), 495(SA), 496(SA), 497(SA), 498(SA), 499(HW), 500(HW), 501(HW)[c], 502(WW), 503(WW), 504(WW)[c], 505(CO), 506(CO)[c], 507(CP), 508(CP), 509(CP), 510(CP), 511(CP).

OCTOBER: 512(SM), 513(SM), 514(SM), 515(SM), 516(SM), 517(PO), 518(SM)[c], 519(IR), 520(IR), 521(IR), 522(IR)[c], 523(AT)[c], 524(SU), 525(SU)[c], 526(EM), 527(EM), 528(EM), 529(EM), 530(EM)[c], 531(EM), 532(EM)[c], 533(PO).

NOVEMBER: 534(HY), 535(HY), 536(HY), 537(HY), 538(HY)[c], 539(ST), 540(ST), 541(ST), 542(ST), 543(ST), 544(ST), 545(SA), 546(SA), 547(SA), 548(SM), 549(SM), 550(SM), 551(SM), 552(SA), 553(SM)[c], 554(SA), 555(SA), 556(SA), 557(SA).

DECEMBER: 558(ST), 559(ST), 560(ST), 561(ST), 562(ST), 563(ST)[c], 564(HY), 565(HY), 566(HY), 567(HY), 568(HY)[c], 569(SM), 570(SM), 571(SM), 572(SM)[c], 573(SM)[c], 574(SU), 575(SU), 576(SU), 577(SU), 578(HY), 579(ST), 580(SU), 581(SU), 582(Index).

VOLUME 81 (1955)

JANUARY: 583(ST), 584(ST), 585(ST), 586(ST), 587(ST), 588(ST), 589(ST)[c], 590(SA), 591(SA), 592(SA), 593(SA), 594(SA), 595(SA)[c], 596(HW), 597(HW), 598(HW)[c], 599(CP), 600(CP), 601(CP), 602(CP), 603(CP), 604(EM), 605(EM), 606(EM)[c], 607(EM).

FEBRUARY: 608(WW), 609(WW), 610(WW), 611(WW), 612(WW), 613(WW), 614(WW), 615(WW), 616(WW), 617(IR), 618(IR), 619(IR), 620(IR), 621(IR)[c], 622(IR), 623(IR), 624(HY)[c], 625(HY), 626(HY), 627(HY), 628(HY), 629(HY), 630(HY), 631(HY), 632(CO), 633(CO).

MARCH: 634(PO), 635(PO), 636(PO), 637(PO), 638(PO), 639(PO), 640(PO), 641(PO)[c], 642(SA), 643(SA), 644(SA), 645(SA), 646(SA), 647(SA)[c], 648(ST), 649(ST), 650(ST), 651(ST), 652(ST), 653(ST), 654(ST)[c], 655(SA), 656(SM)[c], 657(SM)[c], 658(SM)[c].

APRIL: 659(ST), 660(ST), 661(ST)[c], 662(ST), 663(ST), 664(ST)[c], 665(HY)[c], 666(HY), 667(HY), 668(HY), 669(HY), 670(EM), 671(EM), 672(EM), 673(EM), 674(EM), 675(EM), 676(EM), 677(EM), 678(HY).

c. Discussion of several papers, grouped by Divisions.

e. Presented at the Atlantic City (N.J.) Convention in June, 1954.